A SURVEY OF CHAOS AND ITS APPLICATIONS

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ABSTRACT

The relatively new science of chaos has now entered an exciting stage of practical application to real-world problems, impacting such diverse areas as communications, fluid mechanics, and physiology. This paper will introduce the basics of dynamical systems, provide a beginner's guide to chaos and the closely related disciplines of fractals and wavelets, and discuss several novel communications/signal processing applications of interest to microwave engineers.

1. INTRODUCTION

Until only recently, the field of nonlinear dynamics has remained within the confines of academia, and has found limited practical application to engineering problems. However, this situation is now undergoing a revolution of sorts, given (1) the several paradigm-shifting discoveries that have taken place in the closely related fields of chaos, fractals, and wavelets; (2) the advent of powerful computing tools that make the complex numerical simulation of nonlinear phenomena possible; as well as (3) the ever more pressing need to account for, and deal with nonlinear effects that can no longer be adequately handled by mere linear approaches. This paper will provide a top-level introduction and survey of nonlinear dynamics, especially the phenomena of chaos and chaotic synchronization, as well as touch upon the application of nonlinear techniques to the three fundamental considerations in communications design: efficiency, reliability, and security. In particular, such designs seek to maximize information density, be immune to natural and artificial interference, or ensure that the message sent be received or understood by only an authorized listener. The applications presented here will focus on the harnessing of chaos for private/secure communications—illuminating techniques that could rival and replace traditional approaches. Other potential novel applications of chaos will also be proposed, as well as a quick mention of the already well-established uses of fractals and wavelets—both impacting the other two aspects of communications.

2. FUNDAMENTALS OF NONLINEAR DYNAMICS AND CHAOS

The field of dynamics concerns the study of systems whose internal parameters (called *states*) obey a set of temporal rules, essentially encompassing all observable phenomena. This endeavor divides into three subdisciplines, namely:
(1) *applied dynamics*, which concerns the modeling process that transforms actual system observations into an idealized mathematical dynamical system (that is, *state equations* that relate the future states to the past states—usually a set of difference, ordinary differential, or partial differential equations);
(2) *mathematical dynamics*, which primarily focuses on the qualitative analysis of the model dynamical system; and
(3) *experimental dynamics*, which ranges from controlled laboratory experiments to the numerical simulation of state equations on computers.

The state temporal behavior is either viewed as a traditional *time series* (i.e., a given state parameter versus time) or, more usually, in a *phase space* perspective wherein the n system states are plotted against each other in an n-dimensional space with time as an implicit parameter (see Figure 1 for the case $n = 2$, adapted from the excellently illustrated text on dynamics by Abraham and Shaw [1]). The latter framework affords a more natural geometrical setting that possesses an arsenal of analysis tools. A dynamical system is said to be *linear* or *nonlinear* depending on whether the *superposition rule* holds: that is, does the sum of responses to individual stimuli (inputs or initial conditions) equal the single response to the sum of the stimuli? The latter (and more general) class is the subject of this paper, since it leads to a virtual universe of effects (chaos is but one) with potential practical import that is just beginning to be realized.

One of the most well known and potentially useful nonlinear dynamical effects is the bounded, random-like behavior called *chaos*—in essence, "deterministic noise" (see [2], for example). Chaos has been found to occur in a whole myriad of dynamical systems modeling phenomena from astronomy to zoology, and in frequency ranges from baseband to optical. This phenomena, and its closely re-

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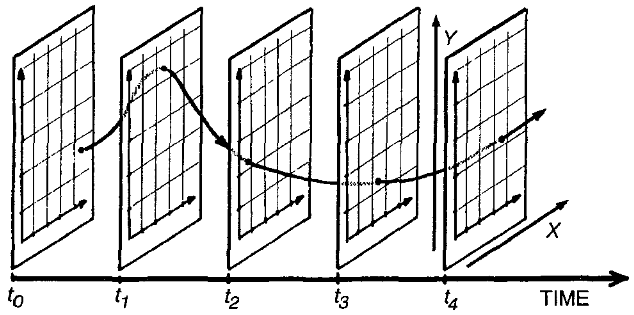

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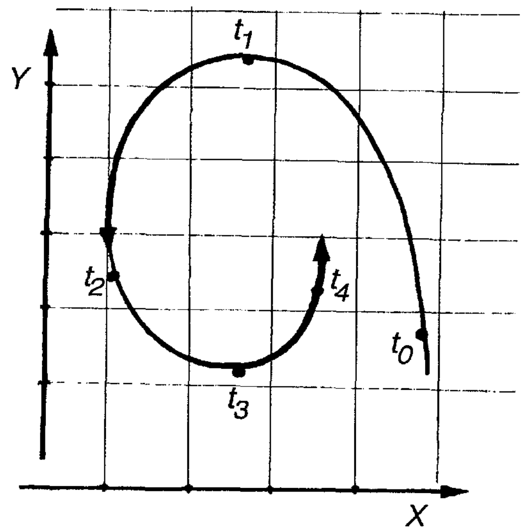

Figure 1: *Two perspectives for representing the temporal evolution of dynamical state variables.* (a) *Time series for a two-dimensional dynamical system.* (b) *Planar phase space orbit obtained by projecting out the time parameter in* (a).

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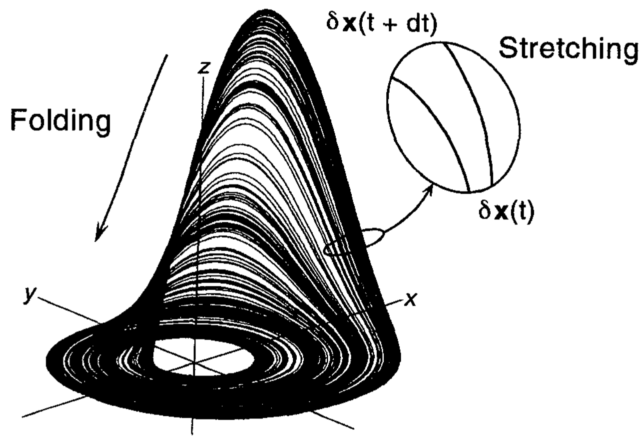

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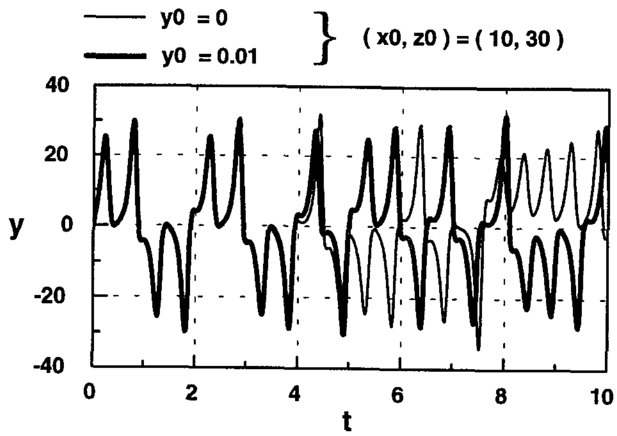

Figure 2: *Some characteristic features of chaotic behavior.* (a) *Chaotic strange attractor from a third-order system.* (b) *Illustration of sensitivity to initial conditions.*

lated *fractal* cousin have been put forth as a new paradigm for understanding and modeling the world around us. This stems from their underlying principle of *self-similarity at different scales* that matches closely with what is observed in nature.

There are three fundamental characteristics of chaos: (1) an essentially continuous and possibly banded frequency spectrum that resembles random noise; (2) *sensitivity to initial conditions*, that is, nearby orbits diverge very rapidly; and (3) an *ergodicity and mixing* of the dynamical orbits which in essence implies the wholesale visit of the entire phase space by the chaotic behavior and a loss of information. Some of these traits are illustrated in Figure 2, whose top portrait shows what is called a *strange attractor*—a primary manifestation of chaotic behavior—in a prototypical third-order, unforced, continuous dynamical system (called the Rössler system). Note the stretch-and-fold operation that is endemic to these attractors, giving rise to their boundedness and fractional dimension (i.e., space-filling nature). Figure 2(b) illustrates the sensitivity to initial conditions for another prototypical third-order chaotic system (known as the Lorenz system). Observe how only a slight change in the initial y-coordinate value y0 leads quickly to very different orbital futures, sometimes referred to as the "butterfly effect." Chaos can be of a transient, intermittent, or steady-state nature, and, in principle, can include an infinite number of both periodic orbits of any period and nonperiodic orbits. These unique properties alone have led to several of the applications listed below.

It was the discovery of *chaotic synchronization* [3] that marked the rapid growth of applied chaos, for it allowed chaos to be modulated and demodulated like a generalized

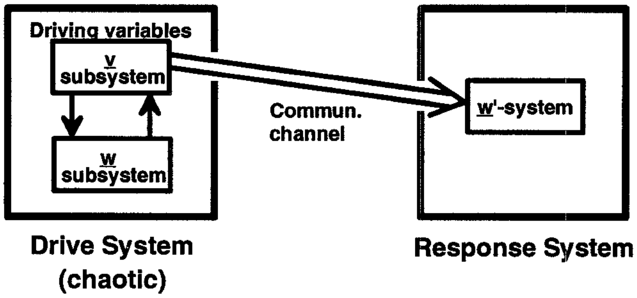

Figure 3: *Schematic illustration of the master-slave form of chaotic synchronization.*

carrier. There are four basic chaotic synchronization techniques already extant: the primary and first *master-slave* form that made chaotic communications possible is depicted in Figure 3 [3]. Here a subsystem of a chaotic system is replicated remotely and driven by the remaining unreplicated state variables. In its simplest continuous instance, the entire drive system would be third-order, while the **w**-, **w**′-subsystem would be second-order, meaning that only a scalar variable v is transmitted across the communications channel linking the drive and response systems. Appropriate rigorous conditions insure the chaotic synchronization of the **w**- and **w**′-subsystems, and they allow for real-world parameter mismatches. This generalized synchronization may prove to possess superior properties compared to its classical digital counterpart (e.g. in its robustness, speed, immunity against channel perturbations and filtering, implementation simplicity, etc.).

3. REPRESENTATIVE APPLICATIONS

The following list enumerates representative applications that have been demonstrated/proposed for chaos, fractals, and wavelets (all of which have self-similarity in common), offering a glimpse of the power and potential of applied nonlinearity:

(i) Employing the natural pseudorandomness of chaotic behavior from nonlinear maps, several chaotic key generators have been formulated for use in traditional digital cryptographic and spread-spectrum systems [4]. Although early versions of this approach were susceptible to short-cycling problems (because of the nature of chaos possessing periodic orbits of all periods), improved modifications have been shown to rival classical feedback shift registers in passing the standard randomness tests.

(ii) In a similar vein, chaotic and quasi-chaotic nonlinear maps (both 1D and 2D) have been used as the basis for data and image encryption [5, 6]. The idea here is that simple nonlinear maps can give rise to very complicated behavior in only a few iterations; and if the process is reversible, then encryption and decryption can be accomplished. The security of the scheme is embedded in the nature of the map and its parameters, the former of which must exactly match between the sender and receiver, while the latter can only allow for very small discrepancies.

(iii) A whole series of baseband communications links have been demonstrated, based on the various forms of chaotic synchronization and modulation that have been developed, the latter ranging from simple additive masking to indirect parameter modulation that could offer enhanced message privacy/security. Figure 4 illustrates one of these setups, using a cascaded form of master-slave synchronization and additive chaotic modulation. This system was found to be quite resilient to noise/interference added to the linking channel, as is needed for a pragmatic communications system. Part (a) of the figure shows the transmitter/receiver configuration that is based on the previously mentioned Lorenz system. In this case, the chaotic carrier x is modulated by adding a voice message at a much lower level, and is recoverable since the chaotic carrier is locally coherently regenerated in the receiver. In part (b), actual experimental results are shown for the chaotic communication system in (a) using baseband speech as the message. Note how the message is buried in the "noise" when viewed in the communications channel, indicating the system's capability for low-probability-of-intercept (LPI) and private transmissions. The author is currently directing efforts at The Aerospace Corporation to realize and explore microwave chaotic communications that would be comparable with current real-world implementations.

(iv) The subfield of *control chaos*, which involves the use of adaptive techniques to parametrically control dynamical behavior, holds much promise in applications ranging from enhanced key generators to chaotic signal constellations [8]. In particular, these techniques allow for the exploitation of the complicated orbits that chaos can harbor, such as keeping the system on long periodic orbits for key generation; or dividing a strange attractor into several regions, each of which can represent a digital symbol, in chaotic signaling.

(v) It has been said by the philosopher Spinoza that there is truly nothing random—what appears random really has an underlying structure that has yet to be discovered. With this motivation, the subdiscipline of *deimbedology* has emerged to try to ascertain the dynamical systems that underlie apparently random processes. This would have great practical import, for if common performance-limiting noise processes could be modeled with chaos (e.g. phase noise in oscillators and amplifiers), they could also be subsequently removed adaptively. This so-called *denoising* is already being accomplished with wavelet techniques in such contexts as musical recordings (decoughing) and medical physiology (smart heart pacemakers).

(vi) Based on the fact that different strange attractors do

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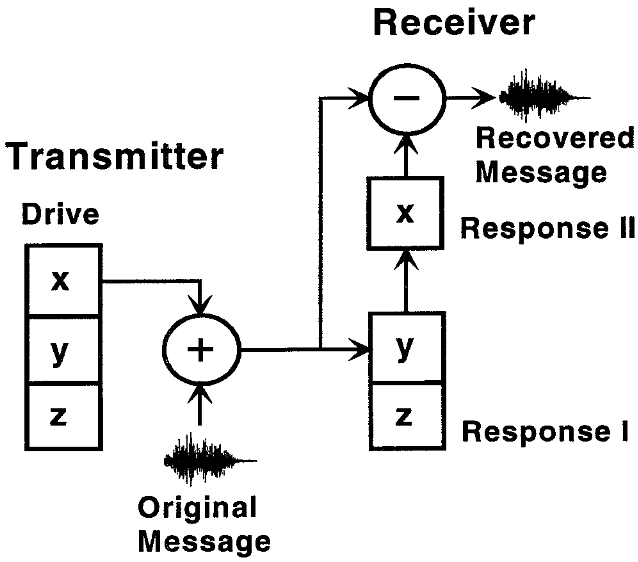

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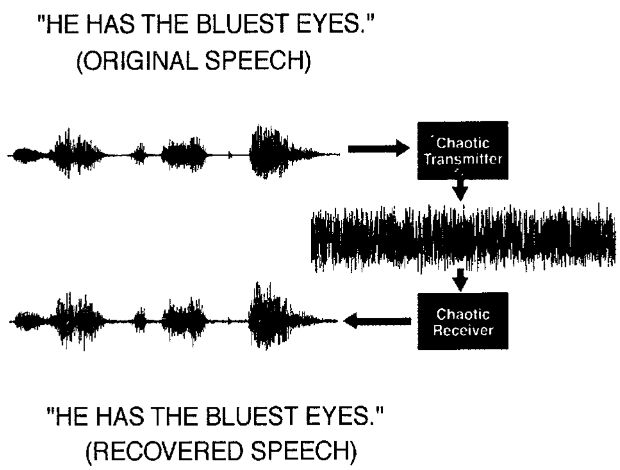

Figure 4: *Example of chaotic masking modulation, one of several means for chaotic communications [7].* (a) *System configuration.* (b) *Experimental results for speech.*

not correlate with each other, analog chaotic versions of CDMA and spread-spectrum systems can also be proposed, with the strange attractor playing the role of a traditional sinusoidal carrier.

(vii) *Fractal interpolation* and wavelet methods have been fruitfully applied to information/image compression, with such examples as the Microsoft® Encarta™ CD and the recently adopted FBI fingerprint database system [9]. These applications are based on the converse of the notion that simple dynamical systems can produce complex behavior.

(viii) A new paradigm for communications signaling is being developed using wavelets, providing generalized redundancy and orthogonality that is effective against such contexts as rapidly changing and unknown channels [10].

4. CONCLUSION

An introduction to the field of nonlinear dynamics and chaos has been presented, along with a set of representative applications. These are a mere sampling of the open frontier of applied nonlinearity—a field that will have a far-ranging impact on future communications systems.

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